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Effects of Nonuniform Stresses and Strains on Measured Characteristic States

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ABSTRACT: Data from triaxial compression tests performed on cylindrical specimens with height-to-diameter $H/D \geq 2$ and with and without lubricated ends as well as experiments with $H/D = 1.0$ and lubricated ends have been analysed to study the effects of experimental techniques on the measured characteristic states. Experiments on four types of sand with several different density indices have been conducted over relatively large ranges of confining pressure. The data shows that the measured characteristic lines are curved when determined from tests with nonuniform stresses and strains, but they are straight when obtained from tests on specimens with $H/D = 1.0$ and lubricated ends, in which the stresses and strains are most likely to be uniform. Determination of the characteristic line is more reliable than measuring the critical state, and it is recommended to involve the former rather than the latter in development of constitutive models and determination of material parameters for such models.

1 INTRODUCTION

The key to understanding the stress-strain behaviour of soils lies in understanding the volumetric changes and, in particular, the factors that influence these. Volume changes can be compressive or expansive in nature. Expansive or dilative volume changes are most pronounced for dense sands at low confining pressures and high stress levels approaching failure. The separation between the region of compression and the region of dilation for drained tests on sand occurs at the characteristic state at which the rate of volume change is zero, $\delta\varepsilon_v/\delta\varepsilon_1 = 0$ (Luong 1980), as shown schematically in Fig. 1. Characteristic states are located on a line, the characteristic line, through the stress origin. The slope of the characteristic line may be described by an angle, φ_{cl}, defined as:

$$\sin\varphi_{cl} = \frac{(\sigma_1 - \sigma_3)_{cl}}{(\sigma_1' + \sigma_3')_{cl}} \tag{1}$$

According to Luong (1980), the resistance is due to pure intergranular friction, i.e. independent of the density index, I_D (formerly known as the relative density, D_r), and the characteristic state is described by an intrinsic parameter, the characteristic angle, φ_{cl} for a given sand. According to Luong (1980) this intrinsic parameter is also independent of the minor principal stress and the stress path.

Lade and Ibsen (1997) studied the characteristic line in view of experimental results from conventional drained triaxial compression tests on

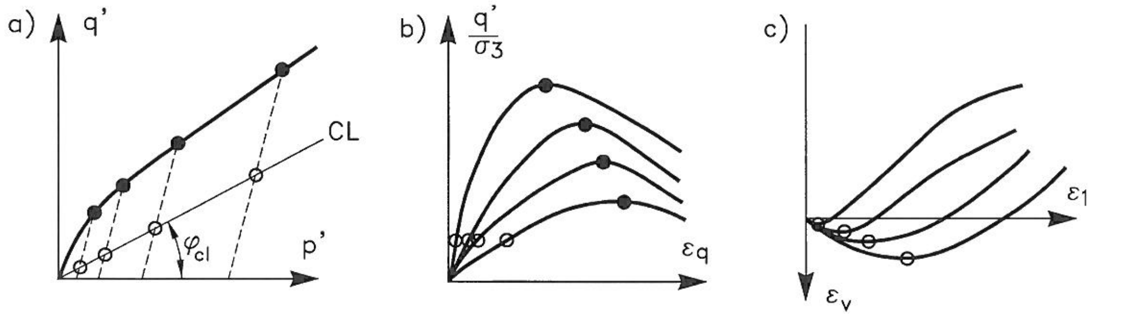

Figure 1. Diagram illustrating the development of stress-strain behaviour in conventional triaxial CD-tests on dense sand performed with different confining pressures on specimens with height-to-diameter ratio H/D= 1.

different sands with a large range of density indices. It was shown that the density index did not influence the location of the characteristic line, and it is therefore unique for a given sand. Most data in this study, performed on specimens with height-to-diameter ratio $H/D > 2$, indicated that the minor principal stress affects the slope of the line. Thus, the characteristic line was found to be curved, with slopes decreasing with increasing confining pressure. The effect of the intermediate principal stress was found to be similar to but less pronounced than its effect on the friction angle.

The characteristic stress state discussed above is that observed in conventional triaxial compression tests in which the stress path corresponds to constant confining pressure and $\delta q/\delta p' = 1/3$. Contraction and dilation can be caused by application of shear stresses and by changes in the mean normal stress. Therefore, the dependency of the characteristic stress state on the stress path has been studied in drained triaxial compression tests by Ibsen & Lade (1998) and by Jacobsen et al. (1999). The conclusion from these studies was that the characteristic line, as originally defined by Luong (1980), is not an intrinsic parameter independent of stress path. Thus, results from tests with different stress paths indicated a significant variation of the characteristic angles φ_{cl}. This variation is due to the fact that the elastic volumetric strain increment varies with mean normal stress. Consequently, the characteristic state was redefined as the stress state where the plastic volumetric strain increment $\delta\varepsilon_v^p = 0$ for the first time. This characteristic state can be measured in p' = constant triaxial compression tests.

The results of studies by Ibsen & Lade (1998) and by Jacobsen et al. (1999) indicate that the characteristic angle must be considered as constant for a given stress path. However, this is in conflict with some of the results presented by Lade & Ibsen (1997), in which the characteristic lines from some experiments were found to be curved, with slopes decreasing with increasing confining pressure.

The difference in results have been identified to be due to different height-to-diameter ratio and different boundary conditions. In the experiments presented by Lade & Ibsen (1997), the specimens had height-to-diameter ratio of $H/D = 2.65$ with and without lubricated ends, whereas the experiments presented by Ibsen & Lade (1998) and by Jacobsen et al. (1999) all were performed on specimens with $H/D = 1$ and with lubricated ends. Presented here is a study of the curvature of the characteristic line, and the relation of the curvature to the nonuniformity in stress and strain, which can develop at very small strains in tall specimens. The influence of the height-to-diameter ratio and the boundary conditions on the measured characteristic state and the stress and strain values at failure are studied in this paper.

2 TEST DATA

Data from previous studies and results of conventional triaxial tests performed by Kjeldsen & Thøgersen (1996) are the bases of the present study.

The latter experiments were performed on reconstituted specimens on Hokksund sand, which is a uniformly graded sand composed of angular grains. The classification properties of the sands are summarised in Table 1. The 54 mm-diameter specimens were prepared by air pluviation with an initial void ratio of 0.563 corresponding to a density index I_D = 93 %. Experiments were performed on saturated specimens with height-to-diameter ratio $H/D = 2.00$ and with and without lubricated ends, and on specimens with $H/D = 1$ and lubricated ends.

3 INFLUENCE OF NONUNIFORM STRESSES AND STRAINS IN TALL SPECIMENS

In conventional triaxial tests the loads, deformations, volume changes or pore pressures are measured outside the specimen. Homogeneous conditions must exist inside the specimen to calculate the correct values of stresses, strains, and void ratios throughout the test. Although it has been advocated since 1965 that triaxial tests should be performed on specimens with lubricated cap and base (Rowe and Barden 1964) and with height equal to diameter (Bishop and Green 1965, Jacobsen 1967, Lade 1982), it is often considered that sufficiently uniform conditions are achieved by using tall specimens with heights greater than or equal to two diameters. Today it is still common to perform triaxial tests with rough cap and base, and the consequence of this practise is briefly described in the following.

3.1 Consequence of nonuniform stresses and strains

If the cap and base are supplied with rough, large porous stones with the same diameter as the specimen, then stiff bodies are created at the ends, as shown in Fig. 2(b). The stress state appears to be very complex with singularities along the edges. The stress state is not homogeneous, because the stiff bodies reinforce the specimen. If tests are performed on a specimen with height equal to diameter, the strength of the soil will be heavily overestimated. This is generally accepted, and tests conducted with rough cap and base are always carried out on tall specimens to minimise the effects of the fixed ends. Whether the material is stable or unstable, the influence of the fixed ends will be measurable, even when the test is carried out on a tall specimen. The specimen will deform as a barrel and only part of the material will actually be sheared.

Table 1. Classification properties for the sands

	Hokksund sand	Aalborg University sand No. 1	Santa Monica Beach sand	Sacramento River sand
Mineral composition	56% quartz 32% feldspar 16 % mica	98% quartz 2% mica	45% quartz 45% feldspar 8% magnetite 2% trace minerals	mostly feldspar and quartz
Particle shape	angular to subangular	angular to subangular	angular to subangular	subangular to subrounded
Specific gravity G_s	2,72	2,64	2,66	2,68
Grain size d_{100}	1,200	0,425	0,425	0,297
size d_{50}	0,400	0,140	0,265	0,240
size d_{10}	0,180	0,085	0,180	0,160
Uniformity coefficient C_u	2,00	1,78	1,58	1,50
Max. void ration e_{max}	0,891	0,854	0,91	1,03
Min void ration e_{min}	0,538	0,549	0,58	0,61

If a homogeneous stress state is to be achieved, the surface load must satisfy the statical condition. The specimen is loaded horizontally by the cell pressure, and the surface is a principal plane, $\tau = 0$, as shown in Fig. 2. To satisfy the statical condition, the surface along the cap and base must be a principal plane too, and this requires that the cap and base must be smooth. This is most often achieved by using a sandwich of thin rubber membranes and silicone grease (Rowe and Barden 1964).

3.2 *Effects of nonuniform stresses and strains on the measurement of failure values and critical states*

The effect of performing tests with rough cap and base can be seen in Fig. 3(a). This diagram shows the load-displacement (F - u_1) and the volume change-displacement (ΔV - u_1) curves from tree triaxial compression tests on dense Hokksund sand with I_D = 93% and confining pressure of 550 kPa. The q - ε_1 and $\varepsilon_v - \varepsilon_1$ curves of the same tests are shown in Fig. 3(b). The stresses and strains are calculated according to traditional analysis of triaxial tests using area corrections and engineering strain expressions for the axial and volumetric strains.

Fig. 3 shows that both the load-displacement and the stress-strain curves, from the test with *H/D* = 2 and fixed ends, are steeper at small strains than those from the two other tests. The strength of the *H/D* = 2 and fixed ends experiment, defined as:

$$\sin \varphi_s = \frac{(\sigma_1 - \sigma_3)_s}{(\sigma_1' + \sigma_3')_s} \qquad (2)$$

is considerably greater compared to the two other tests with free ends, i.e. $\varphi_s = 43.5^0$ compared to $\varphi_s = 40.0^0$ for the *H/D* = 1 specimen, as shown in Table 2. The higher strength is the result of the end restraint imposed by the rough cap and base. It may be noted that the stress-strain curve breaks over much more sharply and the axial strain at failure, ε_1, is considerably smaller, 5.9 %, than for the test performed on the specimen with *H/D* = 1 and free ends for which ε_1 = 10.75 %, as shown in Table 2. These effects are due to the fact that most of the deformation of the specimen occurs only in parts of the specimen, as indicated in Fig. 2(b), and the rest of the specimen deforms very little. By comparing the volume change-displacement curves in Fig. 3(a), it may be observed that even though the specimen with *H/D* = 2 and fixed ends has double volume of the specimen with *H/D* = 1, it compresses less and expands the same as the short specimen with free ends. The main portions of the measured shear deformations and volume changes do not relate to the whole specimen as normally assumed, but only to the parts of the material being sheared. Consequently, the vertical deformation is dominated by the movements in a zone and the strain ought to be calculated relative to the height *h* of this zone and not to the height H_0 of the entire specimen. Unfortunately, the height and the volume of the sheared zone are unknown, so the quantities relating to the entire specimen are always used. The strain measures are therefore wrong and consequently the stress-strain curve will be too short and the volume

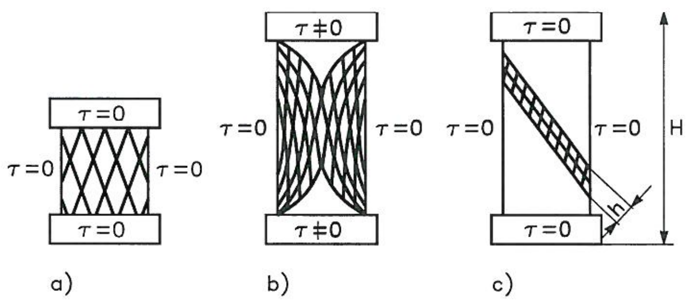

Figure 2. Outline of a triaxial test run. (a) Homogeneous stress and strain conditions, Zone failure occurs. (b) Inhomogeneous stress condition as a result of rough endplates. (c) Line failure can occur under nonuniform strain conditions.

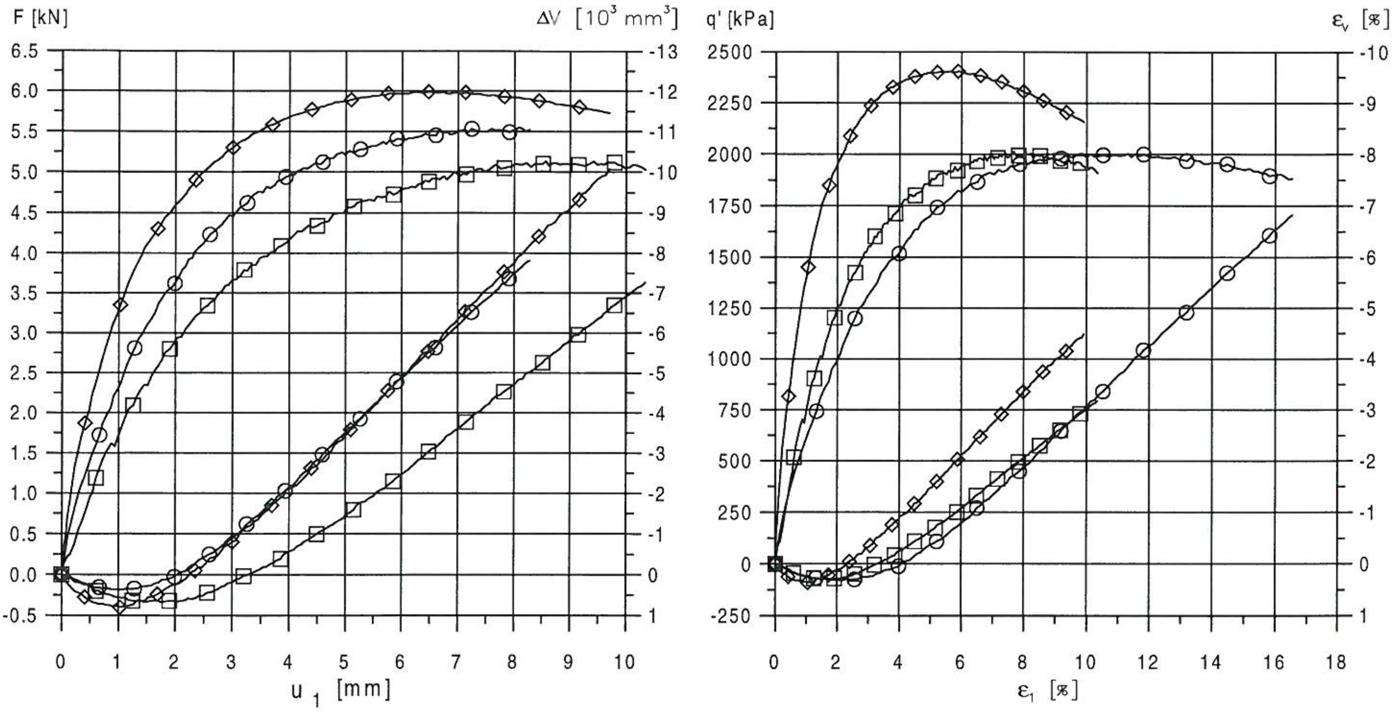

Figure 3. Comparison of: (a) load-displacement and volume-displacement curves in three triaxial compression tests on dense Hokksund Sand I_D = 93 %, confining pressure 550 kPa. (b) q - ε_1 and $\varepsilon_v - \varepsilon_1$ curves of the same tests, calculated according to traditional analysis of triaxial tests using area correction. *H/D* = 2 and fixed ends marked ◇. *H/D* = 2 and free ends marked □. *H/D* = 1 and free ends ○.

change curve too steep, as shown in Fig. 3(b). The angle of dilation, ψ, defined as:

$$\sin \psi = \frac{(\dot{\varepsilon}_v / \dot{\varepsilon}_1)}{(\dot{\varepsilon}_v / \dot{\varepsilon}_1) - 2} \tag{3}$$

is nevertheless determined correctly, as shown in Table 2. This is due to the fact that the rate of dilation, $(\dot{\varepsilon}_v / \dot{\varepsilon}_1)$, is not affected by the incorrectly calculated total strain measures.

By comparing the strengths of the two tests with free ends, see Table 2, it can be concluded that the relative height of the specimen does not play any role in the stress distribution inside a specimen bounded by free end plates. The strength of the two experiments are nearly equal, but severe nonuniformities in strain can develop in unstable material, if the height of the specimen is greater than the diameter. After testing, especially of firm soil, the normal shape of a tall specimen with free ends shows failure as a narrow, ruptured zone, a shear band, where two practically solid bodies slide past each other, as shown in Fig. 2(c). In the experiment with *H/D* = 2 and free ends the effect of the localisation is seen to progress gradually. In the beginning of the tests the stress-strain and volume change-axial strain curves are identical for the two tests. As the shear stresses increase the stress-strain curve of the *H/D* = 2 specimen becomes steeper, and the deformations and volume changes are gradually localised into smaller portions of the specimen. Again, the strain ought to be calculated from the height h of this localised zone and not from the height H_0 of the entire specimen, as shown in Fig. 2(c). The consequences are too steep stress-strain and volume change curves. The stress-strain curve breaks over more sharply and the axial strain at failure, ε_1 = 8.22 %, is considerably smaller, than in the test performed on the specimen with *H/D* = 1 and free ends, ε_1 = 10.75 %, as shown in Table 2.

Because the deformations and volume changes are gradually localised into smaller portions of the specimen, the rate of dilation, $(\dot{\varepsilon}_v / \dot{\varepsilon}_1)$, is changing during the process to failure. As seen in Table 2, this results in an angle of dilation, ψ, for the tall specimen which is smaller than that measured in the test conducted on the specimen with *H/D* = 1. It is also seen that the influence of the localised zone on the measured values φ_s, ψ, ε_1 and ε_v is smaller than the influence of the rough end plates. In Fig. 3(b) the

Table 2. Measured values at characteristic state and failure. Hokksund Sand I_D = 93 %, confining pressure 550 kPa.

		H/D=1 Free ends	H/D=2 Free ends	H/D=2 Fixed ends
Failure	φ_s	40,0	40,2	43,5
	ψ	13,3	10,1	14,0
	ε_1	10,78	8,22	5,87
	ε_v	-3,53	-2,18	-2,03
Characteristic	φ_{cl}	28,7	30,6	34,7
	ε_1	2,08	1,75	1,05
	ε_v	0,31	0,29	0,36

measured stress-strain curve from the H/D = 2 and fixed ends test is seen to have very little to do with the "real" stress-strain curve for the soil and ought not be used as basis for development or calibration of constitutive models. The deviation of the stress-strain curve from the H/D = 2 and free ends test is also so large, especially in terms of the axial strain ε_1 and the ψ measurement, that its relevance in the investigation of the basic phenomena of soil behaviour is questionable.

The characteristic state and the critical state are very similar. Schofield and Wroth (1968) defined the critical states as "states where yield can continue to occur without change in q , p' and ε_v .

$$\left(\frac{\delta p'}{\delta \varepsilon_1}\right) = \left(\frac{\delta q}{\delta \varepsilon_1}\right) = \left(\frac{\delta \varepsilon_v}{\delta \varepsilon_1}\right) = 0 \tag{4}$$

For loose sand and sand at high confining pressure, $\delta\varepsilon_v/\delta\varepsilon_1 = 0$ is reached at the critical state. The critical state is therefore the same as the characteristic state, and it occurs at failure for sand that compresses during shear. For dense sand or sand at low confining pressure, the characteristic state is reached at small strain magnitudes, as indicated by open circles in Fig. 1(c), while the critical state is reached at large strains. If a heterogeneous strain state is permitted to develop, the volume changes will be concentrated in narrow failure zones and only small volume changes occur in the sand mass outside this zone. Once the sand in the failure zone has expanded to the critical void ratio, no further volume change takes place. Fig. 4 shows the results of two drained triaxial tests on dense Santa Monica Beach sand with I_D = 0.9. Index properties are shown in Table 1. The tests are performed with H/D = 2.7 and fixed ends, and H/D = 1 and free ends, respectively. Also in these tests the rough cap and base are seen to cause the specimen to strengthen, shown by development of the steep curve at small strains and the 10% higher failure stress. Fig. 4(b) also indicates that the dilation of the specimen with H/D = 2.7 and fixed ends has essentially ceased at about 12% axial strain, whereas in the specimen with H/D = 1 and free ends, volume changes occur and continue even at 35% axial strain. As the volume change measured in a triaxial test represents the average volume change for the entire specimen, the final average void ratio for the H/D = 2.7 specimen is seen to be much smaller than the actual critical void ratio for the sand, at a given confining pressure.

It is important to notice that the residual strength is approximately 30% larger in tests with H/D = 2.7 than in tests with equal height and diameter.

The tests in Fig. 5 have been performed on Santa Monica Beach sand with a density index I_D = 0.2. The fixed ends influence the measured results, even in loose specimens. In tests performed with fixed ends the curves are steeper at small stresses and the stress at failure is larger than in tests performed with

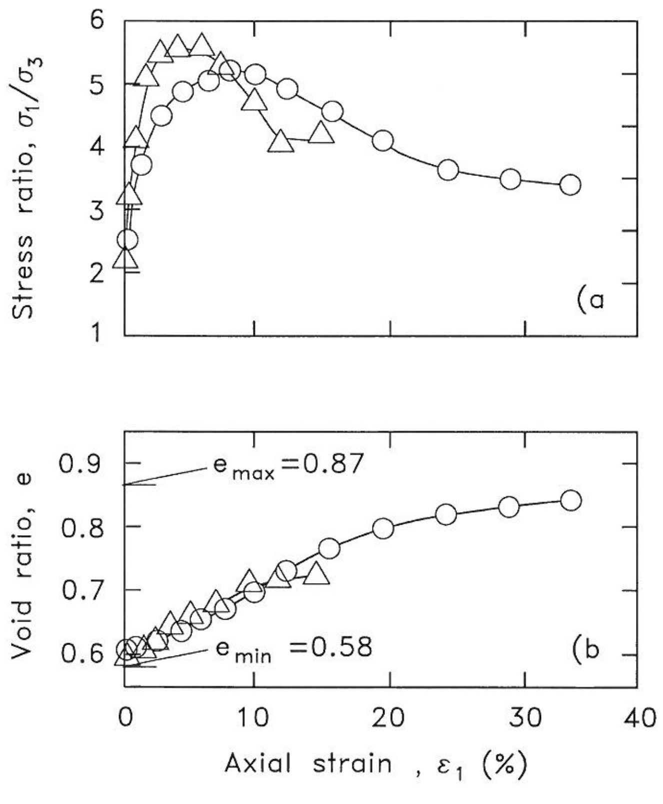

Figure 4. Comparison of stress-strain relations and void ratio changes in triaxial compression test on dense Santa Monica Beach Sand. I_D = 0,9 with H/D = 1 and free ends marked ○. H/D = 2 and fixed ends marked Δ.

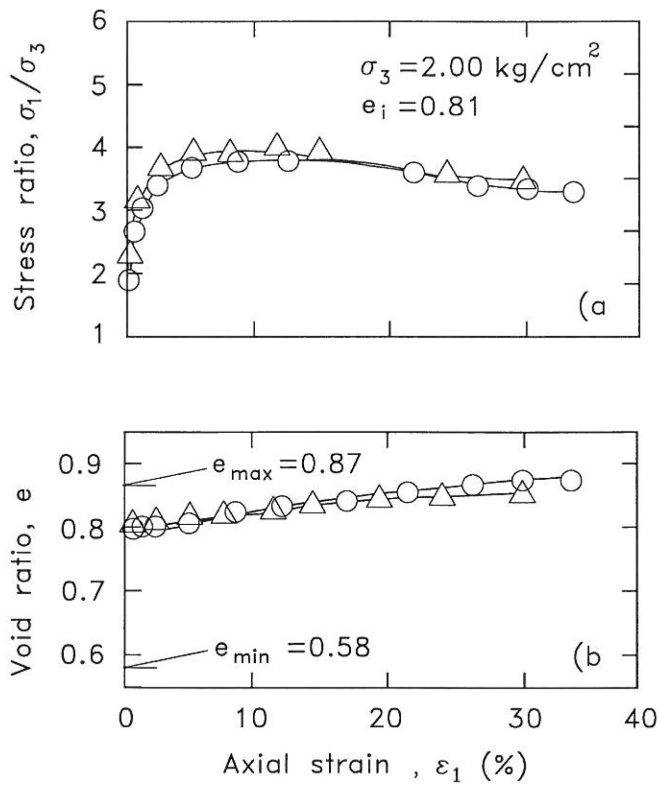

Figure 5. Comparison of stress-strain relations and void ratio changes in triaxial compression test on loose Santa Monica Beach Sand. I_D = 0,2 with H/D = 1 and free ends marked ○. H/D = 2 and fixed ends marked Δ.

free ends. However, the difference is insignificant, and failure is not as clearly defined as in the tests shown in Fig. 4. This is due to the fact that loose sand is a stable soil type, which reduces its volume when the stresses approach failure. If a failure condition starts to develop, the soil will strengthen and the shearing must go elsewhere. Failure is obtained in a uniform condition where all parts carry almost equal shear stress. In this case shear bands do not develop in tests with *H/D* = 2, and the strain-stress curves in Fig. 5 are almost identical for the two tests. Notice that the pattern recurs from the tests performed with dense sand. The reason for the small difference in the two tests in Fig. 5 is the high initial void ratio, which is so close to the critical void ratio that the volume change remains minimal throughout the tests. A comparison between the void ratio in Fig. 4(b) and Fig. 5(b) will show that wherever zone failures occur (obtained by using *H/D* =1 and free ends), they will converge to the same void ratio, namely the critical void ratio. However, specimens with nonuniform stresses and strains result in considerably diverging critical void ratio.

In Fig. 4 it is also seen that even at 35% axial strain the critical state has not yet been fully developed in the specimen deforming under homogeneous stresses and strains (*H/D* = 1 and smooth cap and base). It is questionable if the critical state is at all measurable under homogeneous conditions, because the sandwich, consisting of thin rubber membranes and silicone grease, employed to produce smooth end plates has its limit too. At an axial strain of 35 %, the stretching of the rubber membranes is associated with a non-neglectable force. The membrane force will gradually reinforce the specimen and similar errors as described for the rough end plates will gradually develop at large strain levels.

4 MEASUREMENT OF THE CHARACTERISTIC STATE

The characteristic state can be measured pre-failure and under homogeneous conditions, when employing specimens with *H/D* = 1 and free ends. In comparison, the critical state is measured post-failure at large strain levels and likely inhomogenous conditions. It is consequently advantageous to employ the characteristic angle, φ_{cl}, as a parameter in formulation of constitutive models and/or in parameter determination for such models.

4.1 Measuring the characteristic state

In Fig. 6 the points used to define the characteristic stress states are shown for tests performed on Aalborg University sand No. 1 with I_D = 1.00. The volume changes are determined by weight using a balance with an accuracy of 0.01 g corresponding to a volumetric strains of 0.0037%. This is important in the transition zone, where $\delta\varepsilon_v = 0$, which takes place over 0.2 % axial strain. Due to the stiff response of the soil the transition zone starts at $\varphi = 22.5^0$ and ends at $\varphi = 26.7^0$. In order to measure the characteristic state properly, simultaneous measurements of the axial and volumetric strains at very short time intervals are needed. A time interval corresponding to $\delta\varepsilon_1 = 0.01\%$ and a balance with a precision of 0.005% is required to avoid too much scatter in the measured quantities. The characteristic stress state is considered to occur in the middle of the transition zone. In the experiment shown in Fig. 6, this is seen to correspond to $\varphi_{cl} = 24.6°$.

4.2 Effects of nonuniform stresses and strains on the measurement of characteristic states

As discussed previously the height-to-diameter ratio and boundary conditions have a significant influence on the measured response of the soil. In Fig. 3(b) and Table 2 the measured characteristic state for the same sand with the same density is seen to vary 6^0. The highest characteristic angle is measured in the test producing the largest degree of nonuniform stresses and strains. In the test with *H/D* = 2 and

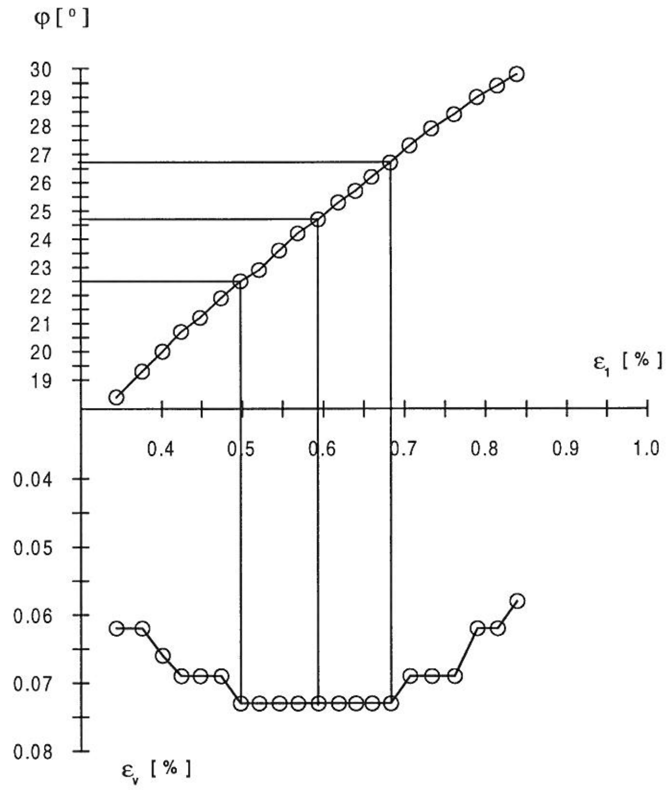

Figure 6. The measured points used to define the characteristic stress state in a triaxial test run on Aalborg University sand No 1. I_D = 1.00.

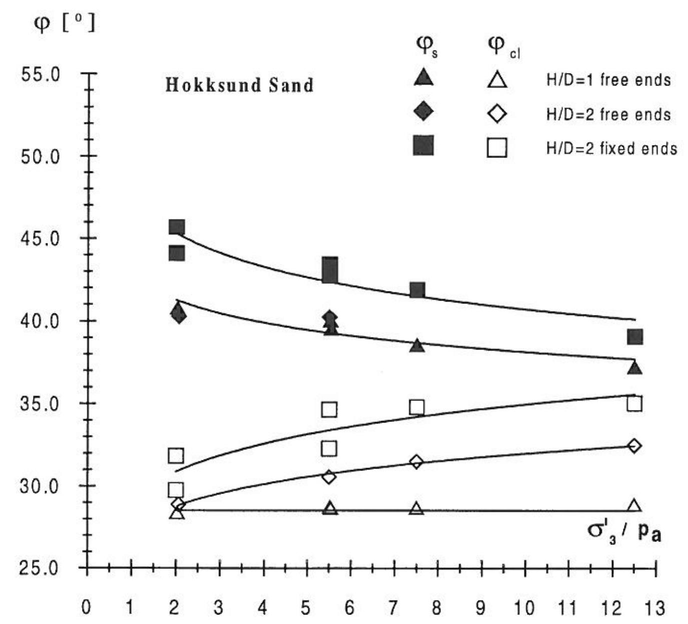

Figure 7. Characteristic angles obtained from triaxial compression tests on Hokksund sand. The tests are performed on specimens with $I_D = 93\%$.

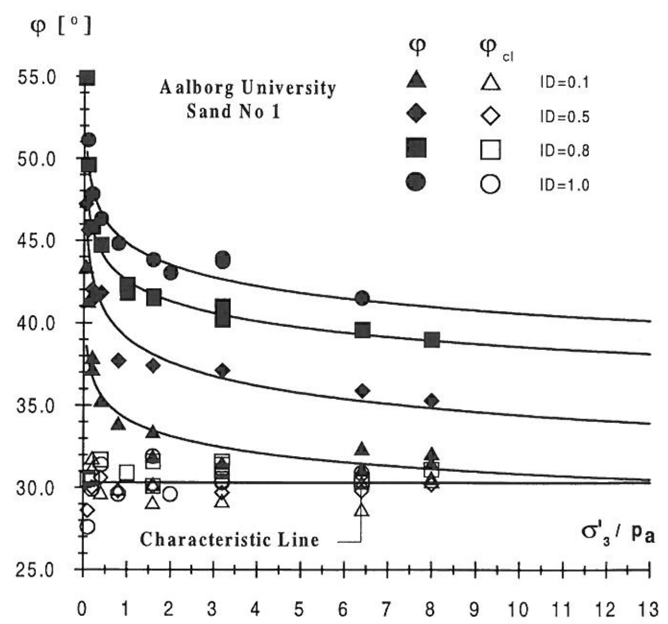

Figure 8. Characteristic angles obtained from triaxial compression tests on Aalborg University sand No. 1.

fixed ends the characteristic angle is measured as $\varphi_{cl} = 34.7^0$, whereas $\varphi_{cl} = 30.6^0$ in the test performed with *H/D* = 2 and free ends, and $\varphi_{cl} = 28.7^0$ in the test with H/D = 1 and free ends. Thus, the characteristic angle is seen to increase with the degree of nonuniformity.

Fig. 7 shows the influence of the minor principal stress. It is seen that the characteristic friction angle is constant and unaffected by the minor principal stress when the tests are performed under homogenous stress and strain conditions. The diagram also shows that the influence of the minor principal stress increases with the degree of nonuniformity. These observations are supported by the test series shown in Figs. 8, 9 and 10. The classification properties of the sands used in these test series are summarised in Table 1.

Fig. 8 shows the characteristic stress states corresponding to triaxial compression tests on Aalborg University sand No. 1 at four relative densities. These experiments were performed on specimens with *H/D* = 1 and with lubricated ends. The experiments show that the characteristic angle φ_{cl} is constant and independent of (1) density index for a given sand, and (2) confining pressure or minor principal stress. As discussed in connection with Fig. 6, the determination of the stress state at which $\delta\varepsilon_v = 0$ is not necessarily very accurate, because the volume change curve is relatively flat near the characteristic state, while the stress state varies considerably. Therefore, the characteristic stress points in Fig. 8 show some scatter.

The characteristic stress states corresponding to triaxial compression tests on Santa Monica Beach sand at four density indices (Lade and Prabucki 1995) are shown in Fig. 9. These experiments were performed on specimens with height-to-diameter ratio of 2.65 and with lubricated ends. They also show that the characteristic angle, φ_{cl}, is independent of density index for a given sand, but due to the development of nonuniformities in strains, the angle is found to increase slightly with the minor principal stress.

The characteristic angles for drained triaxial compression tests on Sacramento River sand (Lee 1965, Lee and Seed 1967) were also determined for four different density indices, and they are shown in Fig. 10. These tests were performed on specimens with D_0 = 3.56 cm and *H/D* = 2.43, without lubricated ends. The data for Sacramento River sand show more scatter than the other test series. Still, the tests indicate that the characteristic angle is independent of density index. Due to the more pronounced nonuniformities in stresses and strains caused by rough end plates, the characteristic angles vary more with confining pressure than the data for Santa Monica Beach sand. It is therefore important to realise that the nonuniform deformations, which develop at very small strains, influence the measured characteristic angles and therefore the conclusions regarding the effect of the minor principal stress.

5 CONCLUSIONS

The characteristic line determined from triaxial compression tests has been shown to become increasingly curved with increasingly nonuniform stress and strain states in the specimen. The best experimental setup involves specimens with *H/D* =

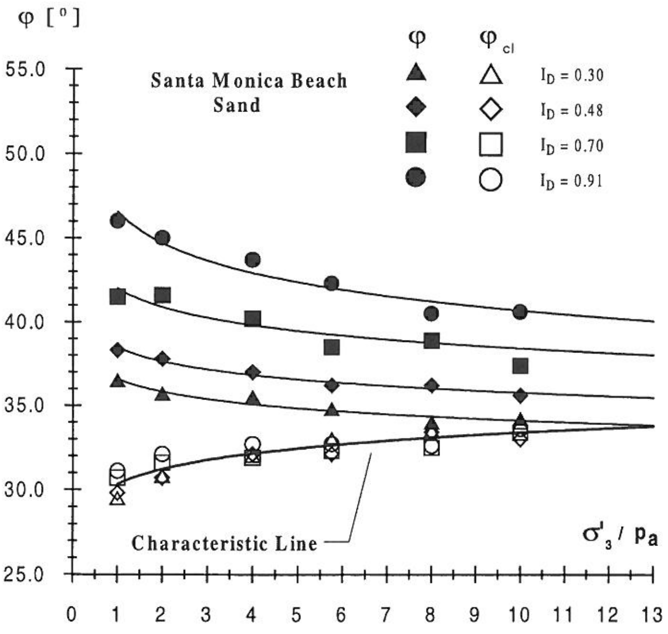

Figure. 9 Characteristic states obtained from triaxial compression tests on Santa Monica Beach sand.

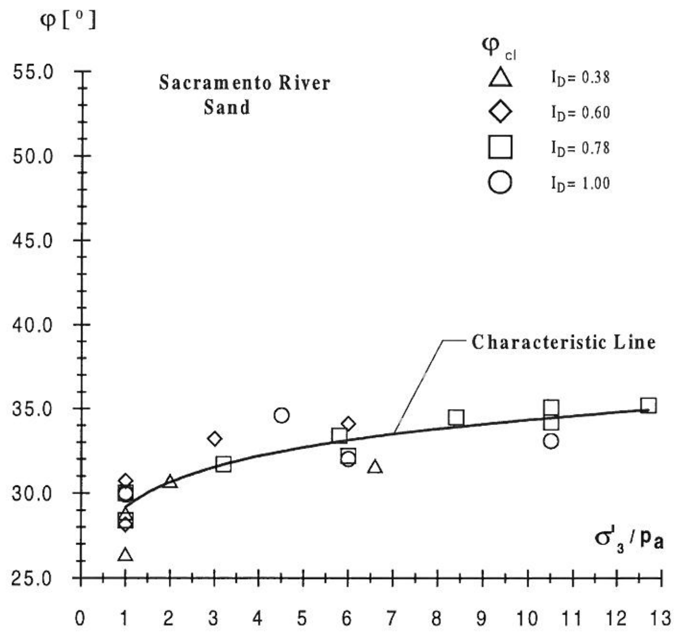

Figure.10 Characteristic states obtained from triaxial compression tests on Sacramento River sand.

1.0 and lubricated ends. This setup is most likely to produce uniform stresses and strains, and experiments performed with this setup show that the characteristic line is straight and not influenced by density index or by confining pressure.

Determination of the characteristic state, which occurs early in a triaxial test, is more reliable than measuring the critical state, because the latter requires very large deformations, and stresses and strains are likely to become nonuniform, even in specimens with *H/D* = 1.0 and lubricated ends. It is therefore advantageous to employ the characteristic angle, φ_{cl}, as a parameter in formulation of constitutive models and/or in parameter determination for such models.

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